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HYDROMAGNETIC COUETTE FLOW OF CLASS-II IN A ROTATING SYSTEM WITH HALL EFFECTS

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Abstract

Steady hydromagnetic Couette flow of Class-II of a viscous, incompressible and electrically conducting fluid between two parallel plates taking Hall current into account in a rotating system is studied. Exact solution of the governing equations is obtained in closed form. The expressions for non-dimensional shear stress at the moving plate and non-dimensional mass flow rates in the primary and secondary flow directions are also derived. Asymptotic behavior of the solution is analyzed for small and large values of rotation parameter K^2 and magnetic parameter M^2 to gain further insight into the flow pattern. Heat transfer characteristics of the fluid flow are considered taking viscous and Joule dissipations into account. The numerical values of fluid velocity and induced magnetic field, computed from the analytical solution, are displayed graphically with respect to channel width variable η for various values of m (Hall current parameter) and K^2 whereas the numerical values of shear stress at the moving plate due to primary and secondary flows and mass flow rates in the primary and secondary flow directions are presented in tabular form for various values of m and K^2. The profiles of fluid temperature are drawn versus channel width variable η for various values of m and K^2 whereas numerical values of the rate of heat transfer at stationary and moving plates are displayed in tabular form for various values of m and K^2.

Index Terms: *Hydromagnetic Couette flow of class-II, Hall current, Coriolis force, modified Ekman boundary layer, modified Hartmann boundary layer, viscous and Joule dissipations.*

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1. INTRODUCTION

Theoretical/experimental investigation of the problems of hydromagnetic flow of a rotating fluid are carried out by several researchers during past few decades due to its considerable importance in various areas of geophysics, astrophysics and fluid engineering. It is widely accepted that terrestrial magnetic field is maintained by fluid motion in Earth's core. A number of astronomical bodies i.e. the Sun, Earth, Jupiter, magnetic stars, pulsars etc possess fluid interiors and (at least surface) magnetic fields. Several important problems of geophysical and astrophysical interest and fluid engineering viz. maintenance and secular variations of Earth's magnetic field due to motion of Earth's liquid core, internal rotation rate of the Sun, structure of the magnetic stars, solar and planetary dynamo problems, turbo machines, rotating MHD generators, rotating drum separators for liquid metal MHD applications etc are directly governed by the action of Coriolis and magnetic forces. It may be noted that in basic field equations the effects of Coriolis force is of much significance as compared to that of inertial and viscous forces whereas Coriolis and magnetic forces are comparable in magnitude. Keeping in view the significance of such topic several researchers [1-20] investigated hydromagnetic Couette flow of an electrically conducting fluid in a rotating system. It may be noted that theory of Couette flow is utilized of the measurement of viscosity and estimating of drag force in many wall driven applications [23]. Taking into consideration the research studies made in the past on hydromagnetic Couette flow, we are of opinion that there are two types of hydromagnetic Couette flow [8, 20, 21, 22], namely, (i) Hydromagnetic Couette flow of class-I and (ii) Hydromagnetic Couette flow of class-II. The fluid flow induced due to the movement of a plate, when fluid is bounded by a stationary plate placed at a finite distance from the moving plate, may be recognized as hydromagnetic Couette flow class-I. This fluid flow is similar to the flow induced due to movement of a plate when free stream is stationary. The fluid flow past a stationary plate, which is induced due to the movement of a plate placed at a finite distance from the stationary plate, may be recognized as hydromagnetic Couette flow of class-II. This fluid flow is similar to the flow past a stationary plate due to a moving free stream. Jana et al. [1], Jana and Dutta [2], Seth and Maiti [3], Seth et al. [4-7], Chandran et al. [9], Singh et al. [10], Ghosh [11], Ghosh and Pop [12], Das et al. [13], Guchhait et al. [14], Jha and Apere [15] and Chauhan and Agarwal [16]

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studied hydromagnetic Couette flow of class-I in a rotating system whereas Seth et al. [8], Singh [17], Hayat et al. [18, 19], and Seth and Singh [20-22] investigated hydromagnetic Couette flow of class-II in a rotating system in the presence of a transverse magnetic field considering different aspects of the problem. It may be noted that in an ionized fluid where density is low and/or the magnetic field is strong, the effects of Hall current become significant as pointed out by Cowling [24]. It plays a vital role in determining the flow features of the fluid flow problems. It is worthy to note that Hall current induces secondary flow in the fluid which is also characteristics of Coriolis force. Therefore, it seems to be important to compare and contrast the effects of these two agencies and also study their combined effects on such fluid flow problems. Taking into account this fact Jana and Dutta [2], Ghosh [11], Ghosh and Pop [12], Guchhait et al. [14], Jha and Apere [15] and Chauhan and Agarwal [16] studied the effects of Hall current on hydromagnetic Couette flow of class-I in a rotating system considering different aspects of the problem.

Aim of the present investigation is to study the effects of Hall current on steady hydromagnetic Couette flow of class-II of a viscous, incompressible and electrically conducting fluid in a rotating system in the presence of a uniform transverse magnetic field taking induced magnetic field into account when the plates of the channel are heated/cooled asymmetrically.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider steady Couette flow of a viscous, incompressible and electrically conducting fluid confined within two non-conducting plates $y=0$ and $y=d$ in the presence of a uniform transverse magnetic field H_0 applied parallel to y-axis. Both the fluid and channel rotate in anti-clockwise direction with uniform angular velocity Ω about y-axis. The fluid flow within the channel is induced due to the movement of upper plate $y=d$ with uniform velocity U_0 in x direction whereas lower plate $y=0$ is kept fixed. Stationary and moving plates of the channel are heated/cooled with uniform temperatures T_0 and T_1 ($T_0<T<T_1$) respectively where T is fluid temperature. Since plates of the channel are of infinite length along x and z directions and flow is steady, so all the physical quantities except pressure are functions of y only.

The fluid velocity $\vec{q}$ and induced magnetic field $\vec{H}$ may be assumed as

$$\vec{q}=\left(u^*,0,w^*\right) \text{ and } \vec{H}=\left(H_x^*,H_0,H_z^*\right), \tag{1}$$

which are in agreement with the fundamental equations of magnetohydrodynamics in a rotating frame of reference. Keeping in view the assumptions made above, the governing equations for the fluid flow problem are given by

$$2\Omega w^*=-\frac{1}{\rho}\frac{\partial p^*}{\partial x}+\upsilon\frac{\partial^2 u^*}{\partial y^2}+\frac{\mu_e H_0}{\rho}\frac{\partial H_x^*}{\partial y}, \tag{2}$$

$$0=-\frac{1}{\rho}\frac{\partial p^*}{\partial y}, \tag{3}$$

$$-2\Omega u^*=-\frac{1}{\rho}\frac{\partial p^*}{\partial z}+\upsilon\frac{\partial^2 w^*}{\partial y^2}+\frac{\mu_e H_0}{\rho}\frac{\partial H_z^*}{\partial y}, \tag{4}$$

$$0=H_0\frac{\partial u^*}{\partial y}+\upsilon_m\frac{\partial^2 H_x^*}{\partial y^2}-m\upsilon_m\frac{\partial^2 H_z^*}{\partial y^2}, \tag{5}$$

$$0=H_0\frac{\partial w^*}{\partial y}+\upsilon_m\frac{\partial^2 H_z^*}{\partial y^2}+m\upsilon_m\frac{\partial^2 H_x^*}{\partial y^2}, \tag{6}$$

where $u^*, w^*, H_x^*, H_z^*, p^*$, ρ, υ, μ_e, υ_m and $m=\omega_e\tau_e$ are, respectively, fluid velocity along x-axis, fluid velocity along z-axis, induced magnetic field along x-axis, induced magnetic field along z-axis, modified pressure including centrifugal force, fluid density, kinematic co-efficient of viscosity, magnetic permeability, magnetic viscosity and Hall current parameter. ω_e and τ_e are cyclotron frequency and electron collision time respectively.

Boundary conditions for fluid velocities are

$$u^*=w^*=0 \quad \text{at } y=0, \tag{7a}$$

$$u^*=U_0,\ w^*=0 \quad \text{at } y=d. \tag{7b}$$

Since plates of the channel are assumed electrically non-conducting so the boundary conditions for induced magnetic fields are

$$H_x^*=H_z^*=0 \quad \text{at } y=0, \tag{8a}$$

$$H_x^*=H_z^*=0 \quad \text{at } y=d. \tag{8b}$$

Equation (3) reveals that modified pressure is constant along y-axis. In hydromagnetic Couette flow of class –I, pressure gradient terms $-\frac{1}{\rho}\frac{\partial p^*}{\partial x}$ and $-\frac{1}{\rho}\frac{\partial p^*}{\partial z}$ are not considered by researchers [1-7, 9-16]. This assumption is valid in view of conditions (7a) and (8a) whereas, in hydromagnetic Couette flow of class-II, pressure gradient terms $-\frac{1}{\rho}\frac{\partial p^*}{\partial x}$ and $-\frac{1}{\rho}\frac{\partial p^*}{\partial z}$ are obtained with the help of boundary conditions (7b) and (8b) which are given by

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x}=0 \text{ and } -\frac{1}{\rho}\frac{\partial p^*}{\partial z}=-2\Omega U_0. \tag{9}$$

Making use of (9) in equations (2) and (4), we obtain

$$2\Omega w^*=\upsilon\frac{d^2u^*}{dy^2}+\frac{\mu_e H_0}{\rho}\frac{dH_x^*}{\partial y}, \tag{10}$$

$$-2\Omega(u^*-U_0)=\upsilon\frac{d^2w^*}{dy^2}+\frac{\mu_e H_0}{\rho}\frac{dH_z^*}{dy}. \tag{11}$$

Introducing the non-dimensional variables

$$\eta = y/d,\ u = u^*/U_0,\ w = w^*/U_0,$$

$$H_x = H_x^*/H_0,\ H_z = H_z^*/H_0, \tag{12}$$

equations (5), (6), (10) and (11), in non-dimensional form, become

$$\frac{d^2u}{d\eta^2} + \frac{M^2}{R_m}\frac{dH_x}{d\eta} - 2K^2 w = 0, \tag{13}$$

$$\frac{d^2w}{d\eta^2} + \frac{M^2}{R_m}\frac{dH_z}{d\eta} + 2K^2 u - 2K^2 = 0, \tag{14}$$

$$\frac{du}{d\eta} + \frac{1}{R_m}\frac{d^2H_x}{d\eta^2} - \frac{m}{R_m}\frac{d^2H_z}{d\eta^2} = 0, \tag{15}$$

$$\frac{dw}{d\eta} + \frac{1}{R_m}\frac{d^2H_z}{d\eta^2} + \frac{m}{R_m}\frac{d^2H_x}{d\eta^2} = 0, \tag{16}$$

where, $K^2 = \Omega d^2/\upsilon$ is rotation parameter which is the reciprocal of Ekman number, $M^2 = \mu_e H_0^2 d^2/\rho\upsilon\upsilon_m$ is magnetic parameter which is square of the Hartmann number and $R_m = U_0 d/\upsilon_m$ is magnetic Reynolds number. The boundary conditions (7a) to (8b), in non-dimensional form, are

$$u = w = 0 \quad \text{at } \eta = 0; \qquad u = 1,\ w = 0 \quad \text{at } \eta = 1, \tag{17}$$

$$H_x = H_z = 0 \quad \text{at } \eta = 0; \qquad H_x = H_z = 0 \quad \text{at } \eta = 1. \tag{18}$$

Representing equations (13) to (16), in compact form, we obtain

$$\frac{d^2F}{d\eta^2} + M^2\frac{db}{d\eta} + 2iK^2F - 2iK^2 = 0, \tag{19}$$

$$\frac{dF}{d\eta} + \frac{d^2b}{d\eta^2} + mi\frac{d^2b}{d\eta^2} = 0, \tag{20}$$

where $F = u + iw,\ b = h_x + ih_z,\ h_x = H_x/R_m$ and $h_z = H_z/R_m$.
The boundary conditions (17) and (18), in compact form, become

$$F = 0 \quad \text{at } \eta = 0; \qquad F = 1 \quad \text{at } \eta = 1, \tag{21}$$

$$b = 0 \quad \text{at } \eta = 0; \qquad b = 0 \quad \text{at } \eta = 1. \tag{22}$$

Equations (19) and (20) subject to the boundary conditions (21) and (22) are solved and the solution for fluid velocity and induced magnetic field is expressed in the following form

$$F(\eta) = \frac{1}{\lambda}\left[A\sinh(\lambda\eta) - B\left(1 - \cosh(\lambda\eta)\right)\right], \tag{23}$$

$$b(\eta) = \frac{1}{\lambda^2}\left[\left(\frac{1-im}{1+m^2}\right)\left\{A\left(1-\cosh(\lambda\eta)\right) - B\sinh(\lambda\eta)\right\} + \lambda(B+\lambda)\frac{2iK^2}{M^2}\eta\right], \tag{24}$$

where $\lambda = \alpha - i\beta$, (25a)

$$\alpha,\ \beta = \frac{1}{\sqrt{2}}\left[\left\{\left(\frac{M^2}{1+m^2}\right)^2 + \left(\frac{M^2m}{1+m^2} + 2K^2\right)^2\right\}^{\frac{1}{2}} \pm \frac{M^2}{1+m^2}\right]^{\frac{1}{2}}, \tag{25b}$$

$$A = \frac{\lambda}{2}\left[\frac{\left(1+m^2\right)2iK^2\lambda\cosh\lambda - M^2(1-im)\sinh\lambda}{\left(1+m^2\right)iK^2\lambda\sinh\lambda + M^2(1-im)(1-\cosh\lambda)}\right], \tag{25c}$$

$$B = -\frac{\lambda}{2}\left[\frac{\left(1+m^2\right)2iK^2\lambda\sinh\lambda + M^2(1-im)(1-\cosh\lambda)}{\left(1+m^2\right)iK^2\lambda\sinh\lambda + M^2(1-im)(1-\cosh\lambda)}\right]. \tag{25d}$$

In absence of Hall current (i.e. m=0) solution (23) to (25) agrees with the solution of Seth and Singh [20].

2.1 Non-dimensional Shear Stress at the plates:

Non-dimensional shear stress components τ_x and τ_z at the moving plate due to primary and secondary flows respectively are given by

$$\left(\tau_x + i\tau_z\right)_{\eta=1} = \frac{\lambda}{2}\left[\frac{\left(1+m^2\right)2iK^2\lambda - M^2(1-im)\sinh\lambda}{\left(1+m^2\right)iK^2\lambda\sinh\lambda + M^2(1-im)(1-\cosh\lambda)}\right]. \tag{26}$$

2.2 Non-dimensional mass flow rates:

Non-dimensional mass flow rates Q_x and Q_z in the primary and secondary flow directions respectively are given by $Q_x + iQ_z = \dfrac{1}{M^2}\left[(1+im)2iK^2 - \lambda B\right]$. (27)

2.3 Asymptotic Behavior of the Solution:

We shall now analyze asymptotic behavior of the solution presented by (23) to (25) for small and large values of M^2 and K^2 to gain further insight into the flow pattern.

Case I: $M^2 \ll 1$ and $K^2 \ll 1$

Since M^2 and K^2 are small, we neglect squares and higher powers of M^2 and K^2 in (23) to (25) to obtain fluid velocity and induced magnetic field distribution as

$$u = \left[\eta + \frac{1}{1+m^2}\frac{M^2}{12}\eta\left(2\eta^2 - 3\eta + 1\right)\right] + \ldots, \tag{28}$$

$$w = -\left[\frac{K^2}{3}\eta\left(\eta^2 - 3\eta + 2\right) + \frac{m}{1+m^2}\frac{M^2}{12}\eta\left(2\eta^2 - 3\eta + 1\right)\right] + \ldots, \tag{29}$$

$$h_x = \frac{1}{2\left(1+m^2\right)}\eta(1-\eta)\left[1 - \frac{\left(1-m^2\right)M^2}{12}\eta(1-\eta) - \frac{mK^2}{6}\left(1 - 3\eta + \eta^2\right)\right], \tag{30}$$

$$h_z = \frac{-1}{12\left(1+m^2\right)}\eta(1-\eta)\left[m\left\{6 - \eta(1-\eta)M^2\right\} + K^2\left(1 - 3\eta + \eta^2\right)\right] \tag{31}$$

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It is revealed from the expressions (28) to (31) that in a slowly rotating system when the conductivity of the fluid is low, primary velocity u is independent of rotation whereas secondary velocity w, primary induced magnetic field h_x and secondary induced magnetic field h_z have considerable effects of Hall current, magnetic field and rotation. This is due to the reason that Hall current as well as rotation induces secondary flow in the flow-field. In the absence of Hall current (i.e. m=0) secondary velocity w and secondary induced magnetic field h_z are unaffected by magnetic field whereas primary induced magnetic field is independent of rotation which is in agreement with the results obtained by Seth and Singh [20].

Case II: K^2 □ 1 and M^2 □ $O(1)$

When K^2 is large and M^2 is of small order of magnitude, fluid flow becomes boundary layer type. For the boundary layer flow near the stationary plate $\eta = 0$, fluid velocity and induced magnetic field represented by (23) to (25) assume the following form.

$$u = 1 - e^{-\alpha_1 \eta} \cos \beta_1 \eta, \tag{32}$$

$$w = -e^{-\alpha_1 \eta} \sin \beta_1 \eta, \tag{33}$$

$$h_x = \frac{1}{2K}\left(\frac{1}{1+m^2}\right)\left[\left\{(1-\eta) - e^{-\alpha_1 \eta}\left(\cos \beta_1 \eta - \sin \beta_1 \eta\right)\right\} + m\left\{(1-\eta) - e^{-\alpha_1 \eta}\left(\cos \beta_1 \eta + \sin \beta_1 \eta\right)\right\}\right], \tag{34}$$

$$h_z = \frac{1}{2K}\left(\frac{1}{1+m^2}\right)\left[\left\{(1-\eta) - e^{-\alpha_1 \eta}\left(\cos \beta_1 \eta + \sin \beta_1 \eta\right)\right\} - m\left\{(1-\eta) - e^{-\alpha_1 \eta}\left(\cos \beta_1 \eta - \sin \beta_1 \eta\right)\right\}\right], \tag{35}$$

where

$$\alpha_1 = K\left\{1 + \frac{M^2(1+m)}{4K^2(1+m^2)}\right\}, \quad \beta_1 = K\left\{1 - \frac{M^2(1-m)}{4K^2(1+m^2)}\right\}. \tag{36}$$

The expressions (32) to (36) demonstrate the existence of a thin boundary layer of the thickness $O\left(\alpha_1^{-1}\right)$ near the stationary plate of the channel. This boundary layer may be recognized as modified Ekman boundary layer and may be viewed as classical Ekman boundary layer modified by Hall current and magnetic field. Expression of α_1 in (36) reveals that α_1 increases on increasing either K^2 or M^2. This implies that the thickness of the boundary layer decreases on increasing either K^2 or M^2. Similar type of boundary layer appears near the moving plate of the channel. The exponential terms in the expressions (32) to (35) damp out quickly as η increases. When $\eta \geq 1/\alpha_1$ i.e. outside the boundary layer region, (32) to (35) assume the following form

$$u \approx 1, \qquad w \approx 0, \tag{37}$$

$$h_x \approx \frac{1}{2K}\left(\frac{1+m}{1+m^2}\right)(1-\eta), \quad h_z \approx \frac{1}{2K}\left(\frac{1-m}{1+m^2}\right)(1-\eta). \tag{38}$$

Expressions in (37) and (38) reveal that, in the central core region, i.e. outside the boundary layer region, fluid flows in the primary flow direction only whereas both the induced magnetic fields persist and vary linearly with η. In the absence of Hall current these results agree with the results of Seth and Singh [20].

Case III: M^2 □ 1 and $K^2 \sim O(1)$

In this case also boundary layer type flow is expected. For the boundary layer flow adjacent to the stationary plate $\eta = 0$, (23) to (25) take the following form

$$u = \frac{1}{2}\left\{a\left(1 - e^{-\alpha_2 \eta}\cos \beta_2 \eta\right) - b e^{-\alpha_2 \eta} \sin \beta_2 \eta\right\}, \tag{39}$$

$$w = -\frac{1}{2}\left\{b\left(1 - e^{-\alpha_2 \eta}\cos \beta_2 \eta\right) + a e^{-\alpha_2 \eta} \sin \beta_2 \eta\right\}, \tag{40}$$

$$h_x = \frac{1}{2M}\left\{\alpha^* R_1 + \beta^*\left(R_2 + S\right)\right\}, \tag{41}$$

$$h_z = \frac{1}{2M}\left\{\beta^* R_1 - \alpha^*\left(R_2 + S\right) + 2\beta_3 \eta\right\}, \tag{42}$$

where,

$$R_1 = R\alpha_3 / M, \; R_2 = mR_1, \; a = 1 + \beta_3 \beta^*, \; b = \beta_3 \alpha^*, \tag{43a}$$

$$R = a\left(1 - e^{-\alpha_2 \eta}\cos \beta_2 \eta\right) - b e^{-\alpha_2 \eta}\sin \beta_2 \eta, \tag{43b}$$

$$S = b\left(1 - e^{-\alpha_2 \eta}\cos \beta_2 \eta\right) + a e^{-\alpha_2 \eta}\sin \beta_2 \eta, \tag{43c}$$

$$\alpha^* = \frac{1}{\sqrt{2}}\left\{\left(1+m^2\right)^{\frac{1}{2}} + 1\right\}^{\frac{1}{2}}, \; \beta^* = \frac{1}{\sqrt{2}}\left\{\left(1+m^2\right)^{\frac{1}{2}} - 1\right\}^{\frac{1}{2}}, \tag{43d}$$

$$\alpha_2 = \alpha_3\left(\alpha^* + m\beta^*\right) + \beta_3 \beta^*, \; \beta_2 = \alpha_3\left(m\alpha^* - \beta^*\right) + \beta_3 \alpha^*, \tag{43e}$$

$$\alpha_3 = M / 1 + m^2, \; \beta_3 = K^2 / M. \tag{43f}$$

It is evident from the expressions (39) to (43) that there arises a thin boundary layer of thickness $O\left(\alpha_2^{-1}\right)$ near the stationary plate of the channel. This boundary layer may be named as modified Hartmann boundary layer and may be viewed as classical Hartmann boundary layer modified by Hall current and rotation. Expressions (43d) to (43f) reveal that the thickness of this boundary layer decreases on increasing M^2. Similar type of boundary layer appears near the moving plate of the channel.

In the central core region, equations (39) to (42) assume the following form

$$u \approx a/2, \qquad w \approx -b/2, \tag{44}$$

$$h_x \approx \frac{1}{2M}\left\{\alpha^* a_1 + \beta^*\left(m_1 a + b\right)\right\},$$

$$h_z \approx \frac{1}{2M}\left\{\beta^* a_1 - \alpha^*\left(m_1 a + b\right) + \beta_3 \eta\right\}, \tag{45}$$

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where $a_1 = a\alpha_3 / M$ and $m_1 = m\alpha_3 / M$.

Expressions in (44) and (45) reveal that, in the central core region, fluid flows in both the primary and secondary flow directions. Primary as well as secondary velocity is affected by Hall current, magnetic field and rotation. Both the primary and secondary induced magnetic fields persist and have considerable effects of Hall current, magnetic field and rotation. Also secondary induced magnetic field h_z varies linearly with η. In the absence of Hall current these results agree with the results of Seth and Singh [20].

2.4 Heat Transfer Characteristics

We shall now analyze heat transfer characteristics of steady hydromagnetic Couette flow of a viscous, incompressible and electrically conducting fluid in a rotating system taking viscous and Joule dissipations into account.

The energy equation is given by

$$\alpha^* \frac{d^2T}{dy^2} + \frac{\upsilon}{C_p}\left[\left(\frac{du^*}{dy}\right)^2 + \left(\frac{dw^*}{dy}\right)^2\right] + \frac{1}{\sigma\rho C_p}\left[\left(\frac{dH_x^*}{dy}\right)^2 + \left(\frac{dH_z^*}{dy}\right)^2\right] = 0, \quad (46)$$

where $\alpha^* = K / \rho C_p$ is thermal diffusivity of fluid.

Boundary conditions for fluid temperature are

$$T = T_0 \quad \text{at } y = 0; \qquad T = T_1 \quad \text{at } y = d. \quad (47)$$

Using the non-dimensional variables defined in (12), equation (46), in non-dimensional form, become

$$\frac{d^2\theta}{d\eta^2} + P_r E_r\left[\left\{\left(\frac{du}{dy}\right)^2 + \left(\frac{dw}{dy}\right)^2\right\} + M^2\left\{\left(\frac{dh_x}{dy}\right)^2 + \left(\frac{dh_z}{dy}\right)^2\right\}\right] = 0, \quad (48)$$

where $\theta = \frac{T - T_0}{T_1 - T_0}$, $P_r = \frac{\upsilon}{\alpha^*}$ and $E_r = \frac{U_0^2}{C_p(T_1 - T_0)}$. θ is non-dimensional fluid temperature, P_r is Prandtl number and E_r is Eckert number.

Equation (48) may be written as

$$\frac{d^2\theta}{d\eta^2} + P_r E_r\left[\left\{\left(\frac{dF}{d\eta}\right)\left(\frac{d\bar{F}}{d\eta}\right)\right\} + M^2\left\{\left(\frac{db}{d\eta}\right)\left(\frac{d\bar{b}}{d\eta}\right)\right\}\right] = 0, \quad (49)$$

where $\bar{F}$ and $\bar{b}$ are the complex conjugates of F and b respectively.

Boundary conditions (47), in non-dimensional form, are

$\theta(0)=0$ and $\theta(1)=1$. (50)

Making use of solution (23) to (25) in equation (49) the resulting differential equation satisfying boundary conditions (50) is solved numerically with the help of MATLAB software. Numerical values of rate of heat transfer at the stationary and moving plates are also computed with the help of MATLAB software for various values of m and K^2.

3. RESULTS AND DISCUSSIONS

To study the effects of Hall current and rotation on the flow field, the numerical values of both the primary and secondary fluid velocities and primary and secondary induced magnetic fields, computed from analytical solution (23) to (25) by MATLAB software, are depicted graphically versus channel width variable η in figures 1 to 4 for various values of m and K^2 taking $M^2 = 36$ whereas that of fluid temperature are displayed graphically versus channel width variable η in figures 5 and 6 for different values of m and K^2 taking $M^2 = 36$, $P_r = 7$, and $E_r = 2$. Figure 1 reveals that primary fluid velocity u decreases near the stationary plate and it increases in the region away from the stationary plate $(i.e.\ 0.25 < \eta \le 1)$ on increasing Hall current parameter m. Secondary fluid velocity w increases in the lower half of the channel and decreases in the upper half of the channel on increasing m. This implies that Hall current accelerates fluid flow in the secondary flow direction in the lower half of the channel and it has a reverse effect on fluid flow in secondary flow direction in the upper half of the channel. Hall current accelerates fluid flow in the primary flow direction in major part of the channel $(i.e.\ 0.25 < \eta \le 1)$. Figure 2 shows that both the primary and secondary velocities increase on increasing K^2. This implies that rotation tends to accelerate fluid flow in both the primary and secondary flow directions. It is apparent from Figure 3 that primary induced magnetic field h_x decreases whereas secondary induced magnetic field h_z increases on increasing m. This implies that Hall current tends to enhance secondary induced magnetic field whereas it has a reverse effect on the primary induced magnetic field.

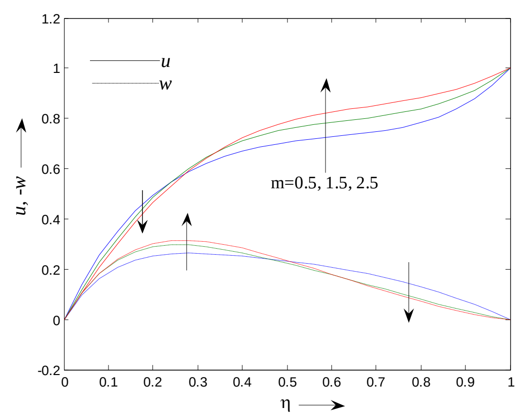

Fig 1: Velocity profiles when $K^2 = 5$.

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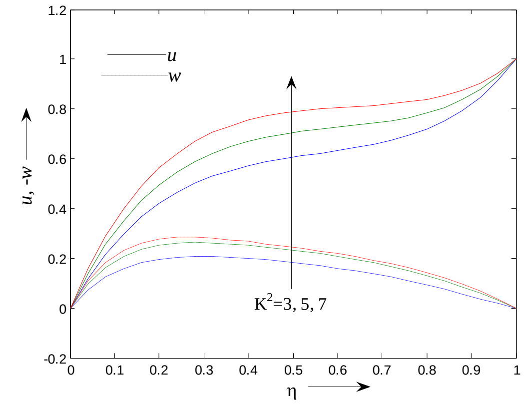

Fig 2: Velocity profiles when $m = 0.5$.

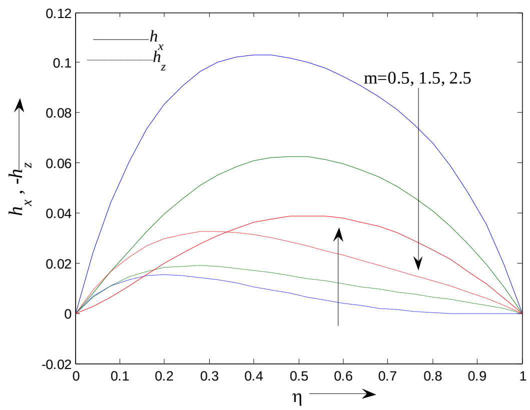

Figure 3: Induced magnetic field profiles when $K^2 = 5$.

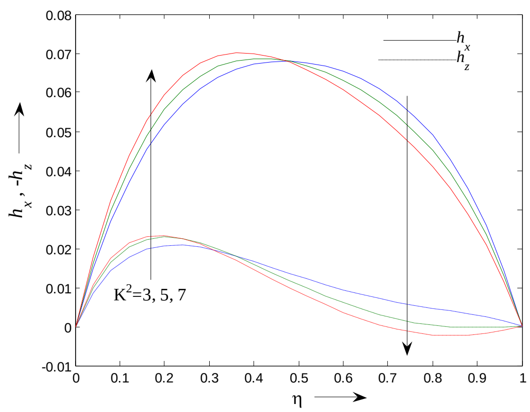

Figure 4: Induced magnetic field profiles when $m = 0.5$.

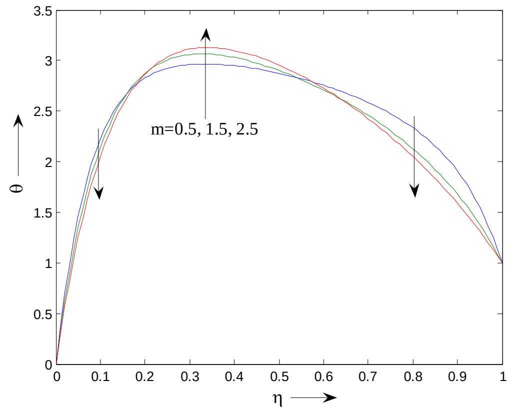

Fig 5: Fluid temperature profiles when $K^2 = 5$.

It is evident from figure 4 that both the primary induced magnetic field h_x and secondary induced magnetic field h_z increase in the lower half of the channel whereas they decrease in the upper half of the channel on increasing K^2. This implies that rotation tends to enhance both the primary and secondary induced magnetic fields in the lower half of the channel whereas it has reverse effect on these induced magnetic fields in the upper half of the

channel. It is noticed from figure 5 that fluid temperature θ decreases in the region near the stationary plate as well as in the region near the moving plate of the channel but it increases in the region $0.2 < \eta < 0.6$ on increasing *m*. This implies that Hall current tends to reduce fluid temperature in the regions near the stationary and moving plates of the channel whereas it has reverse effect on the fluid temperature in the region $0.2 < \eta < 0.6$. It is revealed from figure 6 that θ decreases in the region near the moving plate and it increases in major part of the channel (i.e. $0 \le \eta < 0.8$) on increasing K^2. This implies that rotation tends to enhance fluid temperature in major part of the channel.

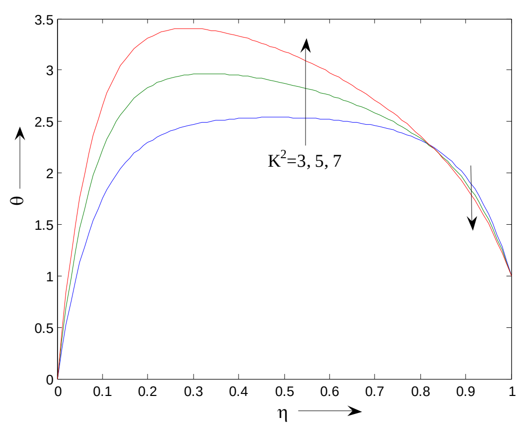

Fig 6: Fluid temperature profiles when $m = 0.5$.

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The numerical values of primary shear stress at the moving plate i.e. $\tau_x|_{\eta=1}$, secondary shear stress at the moving plate i.e. $\tau_z|_{\eta=1}$, primary mass flow rate Q_x and secondary mass flow rate Q_z are computed from the analytical expressions (26) and (27) using MATLAB software and are displayed in tabular form in tables 1 to 2 for various values of m and K^2 taking $M^2 = 36$. Table 1 reveals that $\tau_x|_{\eta=1}$ and $\tau_z|_{\eta=1}$ decrease on increasing m whereas $\tau_x|_{\eta=1}$ decreases and $\tau_z|_{\eta=1}$ increases on increasing K^2. This implies that Hall current has a tendency to reduce both the primary and secondary shear stress at the moving plate whereas rotation tends to reduce primary shear stress at the moving plate and it tends to enhance secondary shear stress at the moving plate. There exists flow separation at the moving plate in the secondary flow direction on increasing m when $K^2 = 3$ and also on increasing K^2 when m=1.5 and m=2.5.

It is found from table 2 that Q_x increases on increasing m whereas Q_z increases when $K^2 = 3$, it decreases, attains a minimum and then increases when $K^2 = 5$ and it decreases when $K^2 = 7$ on increasing m. Q_x and Q_z increase on increasing K^2. This implies that Hall current tends to enhance mass flow rate in the primary flow direction whereas rotation tends to enhance mass flow rate in both the primary and secondary flow directions.

The numerical values of rate of heat transfer at the stationary plate i.e. $d\theta/d\eta|_{\eta=0}$ and rate of heat transfer at the moving plate i.e. $d\theta/d\eta|_{\eta=1}$, computed with the help of MATLAB software for various values of m and K^2 taking $M^2 = 36$, $P_r = 7$ and $E_r = 2$, are presented in tabular form in table 3. It is noticed from table 3 that $d\theta/d\eta|_{\eta=0}$ and $d\theta/d\eta|_{\eta=1}$ decrease on increasing m. $d\theta/d\eta|_{\eta=0}$ increases whereas $d\theta/d\eta|_{\eta=1}$ decreases on increasing K^2. This implies that Hall current tends to reduce rate of heat transfer at both the stationary and moving plates of the channel. Rotation has tendency to enhance rate of heat transfer at the stationary plate where as it has reverse effect on the rate of heat transfer at the moving plate.

4. CONCLUSIONS

An investigation of steady hydromagnetic Couette flow of Class-II of a viscous, incompressible and electrically conducting fluid between two parallel plates taking Hall current into account in a rotating system has been carried out. Significant findings are as follows:

- Hall current accelerates fluid flow in the secondary flow direction in the lower half of the channel and it has a reverse effect on fluid flow in secondary flow direction in the upper half of the channel. Rotation tends to accelerate fluid flow in both the primary and secondary flow directions.
- Hall current tends to enhance secondary induced magnetic field whereas it has a reverse effect on primary induced magnetic field. Rotation tends to enhance both the primary and secondary induced magnetic fields in the lower half of the channel whereas it has reverse effect on these induced magnetic fields in the upper half of the channel.
- Hall current tends to reduce fluid temperature in the regions near the stationary and moving plates of the channel. Rotation tends to enhance fluid temperature in major part of the channel.
- Hall current has a tendency to reduce both the primary and secondary shear stress at the moving plate whereas rotation tends to reduce primary shear stress at the moving plate and it tends to enhance secondary shear stress at the moving plate.
- Hall current tends to enhance mass flow rate in primary flow direction whereas rotation tends to enhance mass flow rate in both primary and secondary flow directions.
- Hall current tends to reduce rate of heat transfer at both stationary and moving plates of the channel. Rotation has tendency to enhance rate of heat transfer at the stationary plate where as it has reverse effect on the rate of heat transfer at the moving plate.

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Table 1: Primary and Secondary Shear stress at the moving plate.

$K^2 \downarrow$ $m \rightarrow$	$\tau_x\|_{\eta=1}$			$\tau_z\|_{\eta=1}$		
	0.50	1.5	2.5	0.50	1.5	2.5
3	2.4183	1.6682	1.2245	0.4044	- 0.0541	- 0.0249
5	1.9793	1.3235	0.9031	0.7572	0.2035	0.1589
7	1.5742	1.0400	0.6653	0.8982	0.2924	0.1920

Table 2: Primary and Secondary mass flow rates.

$K^2 \downarrow$ $m \rightarrow$	Q_x			$-Q_z$		
	0.50	1.5	2.5	0.50	1.5	2.5
3	0.5718	0.5956	0.6056	0.1379	0.1417	0.1506
5	0.6408	0.6697	0.6845	0.1802	0.1731	0.1777
7	0.7016	0.7276	0.7429	0.1943	0.1783	0.1783

Table 3: Rate of heat transfer at stationary and moving plates.

$K^2 \downarrow$ $m \rightarrow$	$\left(\frac{d\theta}{d\eta}\right)_{\eta=0}$			$-\left(\frac{d\theta}{d\eta}\right)_{\eta=1}$		
	0.5	1.5	2.5	0.5	1.5	2.5
3	29.8453	25.3884	22.3012	15.5944	9.8688	7.4899
5	39.2619	34.1276	30.7689	13.6795	8.6286	6.7570
7	47.9343	41.6192	37.8467	12.1884	7.8970	6.4223